Formation of tip-vortices on triangular prismatic-shaped cliffs.

Part 1: A wind tunnel study

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Abstract

Speed-up over topography has long been recognised as significant for structural design and wind energy applications. Here wind flow over cliffs that have a sawtooth plan are studied for speed-up effects under different wind directions. The study undertook PIV measurements to document the mean and turbulence changes in three dimensions for simulated atmospheric boundary layer flow in the University of Sydney BLWT. As the flow approaches the cliffs, it accelerates and moves up to pass the obstruction and generates two strong vortices on the top surface of the cliff. Small changes in wind direction not only lead to significantly different speed-up ratios, but also influence the location of the vortices and the level of turbulence. The complexity of the flow field is revealed in this unique three-dimensional study.

Keywords:
Wind speed-up, steep topography, PIV measurements, Delta-wing vortices

1. Introduction

Speed-up over topography has long been recognised as significant for structural design. From gently sloping 2- and 3-dimensional ridges and hills to vertical cliffs, much effort has been placed on understanding the flow field and providing information for structural designers, including the maximum value of mean and gust speed-up and the extent of the accelerated flow region. In recent times, wind energy applications have focused on locations where enhanced speed can be utilised to increase energy output. Indeed, Wind Energy is one of the fastest growing sources of renewable energy, adding 24\% to its installed capacity in 2009 to reach
159GW worldwide (WWEA, 2010). New wind farms are typically taking advantage of the speed-up effects offered by topography, particularly in coastal regions and exposed hilltops. In West Texas, which became the highest installed wind energy capacity in the US in 2008, escarpment sites were commonly being developed. On the south coast of Australia, many exposed headlands were seen as potentially the best sites for wind farms. But, in a recent review, Ayotte (2010) indicated that such sites can generate substantial turbulence, particularly lateral turbulence that can damage wind turbines and can seriously impact wind energy forecasts.

The aim of this study was to develop a better understanding of the wind speed-up over generic complex topography. The topography chosen here is a 3-dimensional sawtooth plan cliff, one that might be seen in coastal cliffs or inland escarpments. In the longitudinal section the flow sees a step but in plan the flow sees a sawtooth that will concentrate the flow in ‘valley’ regions and form 3-dimensional vortices, not typically seen in a 2-D escarpment. The results presented in this paper are a first step in developing better recommendations for codification of topographic effects and for initial site classifications for wind energy projects. It will also provide much needed data for calibration of numerical models of flow over complex topography.

The flow over a forward-facing step (a 2-dimensional cliff) has been well studied over the years (Castro, 1979; Kiya and Sasaki, 1983). Most recently, the work by Sherry et al. (2010), on the recirculation zone downstream of a forward-facing step, used a similar Particle Image Velocimetry (PIV) technique as the one used in the current study. Conversely, the study of generic steep 3-dimensional topography, such as found on inland escarpments and coastal headlands has received very little attention (Bowen, 1979; Wood, 2000). The vast majority of studies of flow over complex related topography are linked to specific full scale sites (Yamaguchi et al., 2003; Carpenter et al., 2012; Moore et al., 2012).

In the present study, the well documented forward-facing step was first studied as a base case. A generic cliff composed of a set of side by side isosceles triangles was then studied to develop a better understanding of the flow over sawtooth pattern cliffs. Two different triangle shapes were tested for 8 different flow angles. Section 2 presents an overview of the facility and the experimental procedures and Section 3 presents selected results of the fractional speed-up ratio, the pressure coefficient and the turbulence characteristics. Section 4 summarises the results of this study.

2. Facility and Procedures

2.1. Overview

Figure 1 presents a sketch of the 2.5 m wide cliff model installed in the boundary layer wind tunnel of the School of Civil Engineering at the University of Sydney.
The tunnel is an open circuit wind tunnel with a test section of $2.0 \times 2.5$ m and a fetch of 20 meters (Figure 2). The fetch can be equipped with variable levels of roughness to produce different boundary layer profiles with specific turbulence characteristics. The maximum wind speed in the boundary layer section is of the order of 16 m/s. A relatively smooth 1:400 scale atmospheric boundary layer simulation was developed using a trip-board, 4 spires and carpet roughness.

The cliff was composed of a set of side by side right-angled isosceles triangular prisms, referred to as ‘triangles’ hereafter (Figure 1). A reference configuration, a forward-facing step without any triangles, was used for comparison and is referred to as the base cliff configuration in this paper. All of the triangles, except for the middle one, and the base cliff were made from high-density polystyrene and had a height of 100 mm. The three components of the velocity were measured in three horizontal planes over the middle triangle using a stereo-PIV system (David and Gicquel, 2006; Felli et al., 2002; Prasad, 2000). The middle triangle was made from plywood and was equipped with 100 pressure taps on its top surface. The turntable with model attached was rotated to simulate different incoming flow angles. In Figure 1, the flow is moving from left to right and the yaw angle of the cliff is 0°. The coordinate system $xyz$ is fixed relative to the tunnel, with its zero at the centre of rotation of the turntable. In this paper, by convention, the stream-wise direction is along the $x$-axis and the upstream free flow does not have any mean component along the $y$ or $z$-axis.

2.2. Experimental setup

The base cliff spanned the full 2.5 m width of the wind tunnel and was 4.7 m long with an additional 0.9 m long downstream wedge. In Configuration A, 5 triangles were positioned with their long edges along the base cliff, while in Configuration B, the cliff was composed of 7 triangles, with their short edge against the base cliff. Both the pressure and the velocity fields were recorded for each test. The pressures on the top surface of the middle triangle were recorded at 1 kHz for 30 seconds with Honeywell ultra low-pressure differential probes. Tubing correction was applied to retrieve the correct amplitude and phase.

The stereo-PIV system was composed of a TSI double head 200 mJ-per-pulse laser and two 11 MPx cameras. It was setup, in this case, to measure the three components of the velocity on a plane of $0.3 \times 0.2$ m. For each test, 60 statistically independent PIV velocity fields were recorded and averaged to compute the mean flow and the turbulence intensity.

2.3. Experimental procedure

The PIV system was setup to measure the flow profile and boundary layer profile in a vertical plane with and without the base cliff respectively (Figure 3).
Figure 1: Sketch of Configuration A of the cliff model in the wind-tunnel, with the positions of the pressure taps and the two cameras’ view of the PIV windows. The wind flow direction is from left to right. Inserts, clockwise from top left; side view of Configuration A, top view of Configuration A, and, top view of Configuration B.
Flow
Lower hydraulics laboratory
2.4m turntable
Roof of laboratory
Perforated roof
Flow straightener
Sprues, tripods
Test section with cliffs
Carpet roughness
2 m
Working section 2.4m x 2.0m x 20m
Maximum speed 15 m/s

Figure 2: Atmospheric boundary layer wind tunnel at the University of Sydney.

For both Configurations A and B, the PIV system was used to measure the three components of the velocity over the middle triangular cliff:

- at three different heights: 14.3 mm, 17.3 mm and 20.3 mm above the cliff,
- for 9 different incoming flow angles: 0°, 5°, 10°, 15°, 20°, 25°, 30°, 40° and 45°.

To test the symmetry of the tip vortex, an additional angle of 22.5° was used for Configuration B.

For Configuration A, while the measuring plane was set at 14.3 mm above the cliff, three incoming flow speeds ($u_{\text{max}}$ of 7, 9 and 12 m/s) were tested for each angle to ensure that the results were independent of the incoming flow velocity. For all other tests the velocity was constant (9 m/s).

3. Results

3.1. Fractional speed-up ratio, turbulence intensity and pressure coefficients

Except for Section 3.2, all results presented in the following sections are dimensionless. The velocity field was reduced using the fractional speed-up ratio
\( \mathbf{M}(z = h) \), computed as follows for a given height \( h \):

\[
\mathbf{M}(z = h) = (M_u, M_v, M_w)(z = h) = \frac{\mathbf{\nabla}_{\text{Cliff}}(z = h + h_{\text{Cliff}}) - \mathbf{\nabla}_{\text{BL}}(z = h)}{|\mathbf{\nabla}_{\text{BL}}(z = h)|}
\]  

(1)

where \( \mathbf{\nabla}_{\text{Cliff}} \) is the mean velocity measured at a height \( h \) above the cliff, \( h_{\text{Cliff}} \) is the height of the cliff and \( \mathbf{\nabla}_{\text{BL}} \) is the mean velocity of the boundary layer measured at a height \( h \) above the wind tunnel floor using the same PIV system. To simplify the notation, the over-line above \( \mathbf{M} \) was dropped even though the fractional speed-up ratio is defined as a mean value. A fractional speed-up ratio of 1 represents an increase of velocity by a factor 2 over the cliff, a fractional speed-up ratio of 0 denotes no change in velocity and a fractional speed-up ratio of \(-1\) indicates a stagnation point with a velocity of zero over the cliff.

The three fluctuating components of the velocity, \( u' \), \( v' \) and \( w' \) were used to compute the turbulence intensity \( I \):

\[
I = \sqrt{\frac{1}{3} \left( u'^2 + v'^2 + w'^2 \right)} \, |\mathbf{V}|
\]

(2)

where \( |\mathbf{V}| \) is the magnitude of the mean velocity vector.

The pressure coefficient \( C_p \) is the ratio of pressure forces to inertial forces:

\[
C_p = \frac{p - p_{\text{atm}}}{\frac{1}{2}\rho \mathbf{V}^2_{\text{BL}}(z = h_{\text{Cliff}})}
\]

(3)

where \( \rho \) is the air density, \( p \) the absolute pressure on the cliff surface and \( p_{\text{atm}} \) the atmospheric pressure. The pressure transducers record the difference \( p - p_{\text{atm}} \).

3.2. Boundary layer

The mean stream-wise velocity field \( \bar{u} \) in the boundary layer, averaged over 60 flow fields, is shown in Figure 3a, while Figure 4a displays the same velocity field but with the base cliff in place. The two remaining components of the velocity vector \( \mathbf{V} \), \( \bar{v} \) in the cross-wise direction \( y \), and \( \bar{w} \) in the vertical direction \( z \), are not shown here as both \( \bar{v} \) and \( \bar{w} \) are equal to zero in the boundary layer. The boundary layer velocity profile can be approximated with the following logarithm law:

\[
\frac{\bar{u}}{u^*} = \frac{1}{0.4} \ln \left( \frac{z}{z_0} \right) \quad \text{with } z_0 = 0.163 \text{ mm and } u^* = 0.377 \text{ m/s}
\]

(4)

where \( z \) is the height above the wind tunnel floor in millimetres, \( u^* \) is the friction velocity and \( z_0 \) is the roughness height. The turbulence intensity \( I \) decreases from
22%, a couple of millimetres above the surface, to less than 15% at \( z = 250 \text{mm} \). For a typical model at a scale of 1:400, this would represent a category 2 terrain \((z_0 \approx 0.065 \text{ m})\) (AS/NZS 1170.2, 2011).

Unlike Figure 3a, which shows the boundary layer average velocity, Figure 3b reproduces a typical instantaneous velocity field \((u, v, w)\) of the boundary layer. The vectors, overlaying the figures, display the fluctuating velocity components \(u'\) and \(w'\). In Figure 3b turbulence eddies of different scales are clearly visible ranging from the scale of the PIV region of interest, ROI, (Figure 1) window down to a couple of millimetres. Larger eddies are present but cannot be measured by the PIV system in this configuration. In this typical example, the fluctuating components of the velocity are to be compared with the mean velocity shown in Figure 3a and are of the order of 2.5 m/s.

### 3.3. Base cliff

The flow over the base cliff configuration, a forward-facing step, was carried out to validate our setup and compare the obtained results with the triangular configuration. Figure 4 shows the average flow field over the top surface of the base cliff. Figure 4b highlights the recirculation region by showing streamlines within and outside the recirculation region. The flow reattached 250 mm downstream of the face cliff \((2.5 \cdot h_{\text{Cliff}})\), and the maximum height of the recirculation region is 20 mm \((0.2 \cdot h_{\text{Cliff}})\), which is in accordance with the results of Sherry et al. (2010).

The fractional speed-up ratio at \( z = 114.3 \text{ mm}, 14.3 \text{ mm} \) above the base cliff surface, is negative downstream of the cliff edge \((y > 0)\) for at least 350 mm, which means that the wind velocity is lower than the reference one.
3.4. Configuration A

3.4.1. Fractional Speed-up ratio for Configuration A at 0°

The three components of the fractional speed-up ratio $M$ and the turbulence intensity $I$ for Configuration A with an incoming angle of 0° are presented in Figure 5 at a height $h$ of 14.3 mm.

A fractional speed-up ratio $M_u$ (Figure 5a) between 0.5 and 1.0 along the symmetric axis of the triangle demonstrates an increase of the stream-wise velocity of 50 to 100% and could, a priori, justify the installation of wind turbines on the top of the cliff. The wind energy theoretically available to the wind turbine is function of the velocity to the power of 3, the substantial power gain from a fractional speed-up ratio of 0.75 is therefore 5.4. Away from the axis of symmetry of the triangle, the fractional speed-up ratio can have negative values down to $-0.5$, which means that the stream-wise velocity above the cliff is half the velocity of the free stream at the same relative height $h$. Clearly these are not good positions for wind turbines as the power output is significantly reduced.

A comparison with the fractional speed-up ratio $M_u(z = 14.3$ mm) above the base cliff (Figure 4b) shows that the triangle configuration display higher values of $M_u$ for the region of interest.

A closer look at $M_v$ and $M_w$ (Figures 5b and 5c) shows the formation of two delta-wing tip vortices on the upper surface of the triangular cliff. A high level of turbulence is associated with these two regions (Figure 5d). The flow over the surface of the triangular cliff can be divided into almost three equal zones of 30°;
two zones on the sides where the vortices grow and one zone in the middle where
the flow remains in the stream-wise direction $x$. The two vortices are encompassed
by two $30^\circ$ cones.

![Fractional speed-up ratio vector field](image)

(a) $M_u$ in the stream-wise direction. (b) $M_v$ in the cross-stream direction, (c) $M_w$ in
the vertical direction and (d) $I$ the turbulence intensity. (White regions indicate
unavailable data.)

The flow over the triangular shaped cliffs is similar to the flow over a delta-wing,
(Johari and Moreira, 1998). As the flow approaches the cliffs, it accelerates and
moves up to pass the obstruction. $M_w$ clearly shows this upward flow upstream
of the cliff with a maximum upward velocity at the cliff edge (Figure 5c). Once
the wind has passed the edge, the two vortices start to form and grow in a cone of
$30^\circ$. $M_v$ displays the growth of the vortex cone both in the vertical and horizontal
axis (Figure 5b). Close to the tip of the triangle the 14.3 mm high PIV plane only records an inward $M_v$ flow; hence in this region the laser plane is at the top edge of the vortices. Closer to the downstream end of the triangle, $M_v$ is of opposite sign indicating an outward flow (from the centre of the triangle to the edge); the laser plane is now in the lower part of the vortices. This is confirmed by the PIV measurements made at 17.3 and 20.3 mm above the cliff surface, as they are in the top part of the vortex, they do not record an outward velocity (Figure 6b and 6c). Tests performed with different upstream velocities gave similar results.

Figure 6: Fractional speed-up ratio vector field ($M_u, M_v$) and contour map of the fractional speed-up ratio $M_v$ in the cross-wise direction for Configuration A with an incoming flow angle of 0°, (a) at a height $h =$14.3 mm, (b) at a height $h =$17.3 mm and (c) at a height $h =$20.3 mm. (White regions indicate unavailable data.)

A careful examination of Figures 5 and 6 reveals a flow asymmetry; the two vortices are different, with the right one slightly bigger than the left one. In the
right vortex, the transition from an inward (from the cliff edge to the centre) flow to an outward one appears between 20 to 30 mm further upstream than in the left one. The small asymmetries resulted from distortion from horizontal of the turntable, as further detailed in the accompanying paper (Montlaur et al., 2012).

With the three components of velocity and turbulence intensity, the amplitude of the fractional speed-up ratio and the gust (or maximum) fractional speed-up ratio can be computed. The gust fractional speed-up ratio is computed by replacing $\mathbf{V}_{\text{Cliff}}$ by $\mathbf{V}_{\text{gust}}$ in Equation 1. The gust wind speed seen during a gust $V_{\text{gust}}$ was computed using the AS/NZS 1170 approach (AS/NZS 1170.2, 2011): 

$$|V_{\text{gust}}| = |\mathbf{V}| \left(1 + k \cdot \sqrt{3 \cdot I^2}\right)$$

where $k = 3.7$. Figures 7a and 7b display the mean fractional speed-up ratio amplitude and the gust fractional speed-up ratio, respectively. The gust fractional speed-up ratio gives an overall indication of the strength design requirement for a structure of wind-turbine. During a gust the load can almost double.

A wind turbine positioned in one of these vortices would face not only a large velocity gradient, as the amplitude and direction of the incoming flow vary, but also a strong level of turbulence.

3.4.2. Fractional speed-up ratio for Configuration A at 10°

In this configuration, the turntable was rotated anticlockwise by 10°, resulting in a free stream direction of 10° from the right. The windward vortex increases in size, both vertically and horizontally, while the leeward one reduces, compared with
the configuration at 0° (Figure 8). The larger windward vortex is encompassed in a cone of 45-50°, with the smaller leeward vortex in a cone of 25-30°. The level of turbulence increases with the increase in size of the windward vortex, and is reduced in the leeward one.

The windward side vortex axis is positioned at 35° with respect to the normal of the wind direction while the leeward side is at 55°. The windward side presents a larger (projected) surface to the wind than the leeward side, which results in a stronger upward wind velocity (Figure 8c). The stronger upstream velocity results in a stronger and larger cone vortex. Banks and Meroney (2001) found similar results for a corner flow above a flat roof. Figure 8a and 8b show the fractional speed-up ratio $M_u$ and $M_v$ and compare the vortex size.

A wind turbine within the middle zone, between the vortices, would see a fractional speed-up ratio of 0.6 potentially increasing its power output by a factor of 4 (Figure 8a). However, such a position in the middle zone is unsustainable as the zone is narrow and close to the vortex downward flow. If a blade crosses the vortex edge, it will encounter a strong velocity gradient, both in magnitude and direction although the turbulence intensity is unchanged from the upstream conditions.

3.4.3. Pressure coefficient for Configuration A at 0 and 10°

The mean pressure coefficient $C_p$ (Equation 3) on the top surface of the triangle for 0 and 10° is shown in Figures 9a and 9b, respectively. The surface can be, as for the fractional speed-up ratio, divided into three zones: two close to each edge of the cliff, where the vortices are positioned, and one in between. For the case of an incoming flow at 0° the pressure coefficient is symmetric. No asymmetry can be observed in the pressure coefficient, as the pressure taps were positioned on a grid with a mesh spacing of 25mm, which is too large to record the asymmetry noted in the velocity field (Figure 5). For an incoming flow of 10° the windward zone becomes larger and the leeward zone narrower as from the flow field. Even if the asymmetry between the two vortices cannot be recorded with the pressure measurements, they are consistent with the wind speeds measured above the cliff surface.

The pressure above the triangular cliff is very similar to the pressure measured by Banks et al. (2000) on a flat roof subject to corner flow. The minimum measured pressure is located at the lowest point of the recirculation cone where the flow close to the surface is moving upstream with the maximum velocity.

3.4.4. Vortex evolution as a function of the incoming flow angle

As the yaw angle increases, the windward vortex increases and the leeward decreases in size and strength. The windward vortex increases faster than the leeward decreases. At, and beyond a flow angle of 30°, the windward vortex starts
Figure 8: Fractional speed-up ratio vector field \((M_u, M_v)\) for Configuration A with an incoming flow angle of 10° at a height \(h\) of 14.3 mm and contour map of, (a) \(M_u\) in the stream-wise direction, (b) \(M_v\) in the cross-wise direction, (c) \(M_w\) in the vertical direction and (d) \(I\) the turbulence intensity. (White region indicate unavailable data.)
to draw air from the leeward side of the triangle. The evolution of the vortices is displayed in Figure 10 for incoming flow from 0 to 30°. For a yaw angle of 20° the two vortices are clearly visible and distinct, but, for an angle of 30°, the two vortices interact with each other as explained in detail in section 3.4.5.

3.4.5. Fractional speed-up ratios for Configuration A at 40°

A very interesting flow appears for angles between 30° and 40°. An example is given in Figure 11 for a yaw angle of 40°. The windward vortex covers a zone of more than 60° of the triangle (Figures 11a and 11b) while the leeward one has almost disappeared. The relatively low upward velocity on the leeward edge of the triangular cliff is not sufficient to sustain a strong vortex (Figure 11c). The windward vortex, by comparison, is relatively strong.

At the level of the laser sheet, the flow direction of the leeward vortex is toward the centre, while the flow direction of the windward vortex if away from the centre. In the measurement plane, the airflow is from left to right, the consequence of this arrangement is that the windward vortex absorbs the air flow out of the leeward one, even if the two vortices are not connected to each other. Figure 11b shows positive values, up to 1.0, of the fractional speed-up ratio $M_v$ from the leeward vortex toward the windward one. The middle zone is transformed into a channel and is clearly visible in Figure 11a, with a value of $M_v \approx 0.25$.

Except near the windward edge of the cliff and the channel mentioned above, the fractional speed-up ratio $M_u$ in the stream-wise direction is zero or negative (Figure 11a); the velocity, 14.3 mm above the cliff, is lower than in the free stream. The turbulence reaches levels above 100% in both vortices (Figure 11d). In this configuration, no position is ideal for a wind turbine as the turbulence level and the strong velocity gradient would disrupt performance significantly.
Figure 10: Fractional speed-up ratio vector field \((M_u, M_v)\) and contour map of the cross-wise fractional speed-up ratio \(M_v\) for Configuration A at a height \(h\) of 14.3 mm, for an incoming flow of, (a) 0°, (b) 5°, (c) 10°, (d) 15°, (e) 20° and (f) 30°. (White regions indicate unavailable data.)
Figure 11: Fractional speed-up ratio vector field \((M_u, M_v)\) for Configuration A with an incoming flow angle of 40° at a height \(h\) of 14.3 mm and contour map of, (a) \(M_u\) in the stream-wise direction. (b) \(M_v\) in the cross-wise direction, (c) \(M_w\) in the vertical direction and (d) \(I\) the turbulence intensity. (White regions indicate unavailable data.)
3.5. Configuration B

3.5.1. Fractional speed-up ratios for Configuration B at 22.5°

For the free flow direction to be aligned with the axis of symmetry of the triangular cliffs in Configuration B, the turntable had to be rotated by 22.5° as the upstream angle of the cliff had an included angle of 45°. The results presented in Figure 12 for Configuration B at 22.5° are to be compared with Configuration A at 0° described in section 3.4.1. In Configuration B, the upstream angle of the triangular cliff is 45°, while it was 90° in Configuration A.

As in Configuration A, two vortices form at the cliff top and divide the region into three zones. The two vortices are enclosed in 20° cones and leave only a narrow middle zone of about 5°. The fractional speed-up ratio $M_u$ in the streamwise direction is around 0.8, which would reflect a theoretical increase of wind energy of 5.8. The positioning of a wind turbine on the symmetric axis of this triangle is even more critical than in Configuration A as the two vortices would be far closer to the turbine. The turbulence level is lower than in Configuration A as the vortices are smaller.

The fractional speed-up ratio $M_v$ (Figure 12b), in the cross-wise direction, displays negative values in the upstream region; the flow is moving from right to left, due to the yaw angle of the cliff setup. In the downstream region, where adjacent prisms join, the flow is asymmetric; the adjacent prism on the right is further upstream and therefore influencing the flow on the reference cliff, while the prism to the left does not, as it is downstream of the reference cliff. Above the cliff, the flow displays the same behaviour of that in Configuration A; the flow shows an inward direction (toward center) for the first 150 mm in the flow direction, then a transition appears and the flow reverses to outward (away from the center). In the first 150 mm from the cliff tip, the laser sheet cuts the upper part of the vortex and sees an inward flow, while further downstream it transects the lower part of the vortex and sees an outward flow.

3.5.2. Evolution of the fractional speed-up ratio for Configuration B

The evolution of the tip vortices in Configuration B is similar to Configuration A; as the yaw angle increases, the windward vortex increases and the leeward one decreases until the windward vortex draws some air from the leeward side. This phenomena appears at around 10°, 12° away from the symmetric case, which is relatively the same angle as for Configuration A, approximately 1/4 of the apex angle.

Figure 13 shows the evolution of the fractional speed-up ratio $M_v$ as a function of the yaw angle. In this example, the turntable was rotated clockwise and the windward vortex is on the left hand side of the cliff and the leeward one is on the right hand side.
Figure 12: Fractional speed-up ratio vector field \((M_u, M_v)\) for Configuration B with an incoming flow angle of 22.5° at a height \(h\) of 14.3 mm and contour map of the fractional speed-up ratio, (a) \(M_u\) in the stream-wise direction, (b) \(M_v\) in the cross-wise direction, (c) \(M_w\) in the vertical direction and (d) \(I\) the turbulence intensity. (White regions indicate unavailable data.)
Figure 13: Fractional speed-up ratio vector field \((M_u, M_v)\) and contour map of the cross-wise fractional speed-up ratio \(M_v\) for Configuration B at a height \(h\) of 14.3 mm, for an incoming flow of, (a) 22.5°, (b) 15°, (c) 10° and (d) 5°. (White regions indicate unavailable data.)
4. Discussion

Wind tunnel tests were performed to study the evolution of the two tip vortices on generic triangular shape cliffs in a boundary layer flow. The three components of the velocity and the surface pressure were measured for more than 75 tests to assess the influence of:

- the triangle shape; with two configurations tested with apex angles of 90° and 45° respectively. Both configurations give similar results; two delta-wing vortices formed in the flow at each side of the cliff top.

- the yaw angle; with 9 angles tested ranging from 0° to 45°. The yaw angle is the major parameter influencing the vortex flow field, enhancing the vortex on the upwind side of the triangle.

Table 1 summarises some of the results, 14.3 mm above the cliff top surface, by presenting for each configuration studied, the relative area of positive $A(M_u^+)$ and negative $A(M_u^-)$ fractional velocity in the stream-wise direction as well as the average $\overline{M_u}$ in these areas. It is worth noticing that, in Configuration A, even if the area of positive $A(M_u^+)$ decreases with increasing yaw angle, the mean relative speed-up ratio $\overline{M_u^+}$ increases up to 15 °. Above yaw angle of 15 ° the value of $M_u^+$ decreases. The turbulence intensity $I$ increases in both configurations with the yaw angle, but more dramatically in Configuration A than B. For both configurations the average turbulence intensity $\overline{I}$ increase with increasing yaw angles. For Configuration A this increase in turbulence intensity ranges from 30% to 100%, while for Configuration B, it increases from 24% to 31%.

For each test, two delta-wing vortices formed in the flow at each side of the cliff top. The strength of each vortex depended on the yaw angle;

- when the yaw angle is 0°, the flow over the cliff is symmetric and, as expected the two vortices are symmetric and cover approximately one third of the triangular top surface. A stream-wise fractional speed-up ratio $M_u$ up to 100% is observed, in the channel between the two vortices, on the axis of symmetry of the triangle. A fractional speed-up ratio of 100% represents a theoretical wind energy increase of 8.0. Away from the axis of symmetry of the triangle, in the vortex region, the fractional speed-up ratio $M_u$ drops down to -0.5, while the turbulence intensity $I$ rises to value of more than 0.5. A wind-turbine positioned in one of these vortex regions would not only result in less energy produced, as the airflow is slower, but would also add significant loading due to turbulence.

- when the yaw angle is away from 0° and up to ∼ 1/4 of the apex angle (∼ 25° in Configuration A, and ∼ 12° in Configuration B) the windward vortex
Table 1: Comparison of: $A(M_u^+)$, the relative area $A$ of positive value of $M_u$, $M_u^+$, the average value of $M_u$ in the the area $A(M_u^+)$, $A(M_u^-)$, the relative area $A$ of negative value of $M_u$, $M_u^-$, the average value of $M_u$ in the the area $A(M_u^-)$ and $I$ the turbulence intensity as a function of the yaw angle and configuration at 14.3 mm above the cliff top surface.
grows in size and strength, while the leeward vortex reduces. The channel
between the two vortices reduces in size as the yaw angle increases. For a
wind-turbine to see increased power production it needs to be positioned
between the two vortices.

- when the yaw angle is larger than \( \sim 25^\circ \) in Configuration A, and \( \sim 12^\circ \) in
  Configuration B, the windward vortex generates a significant low pressure
  that draws the airflow from the leeward side to the windward side of the
  triangle, leading to a reduced leeward vortex.

A parametric investigation of the influence of the roll and pitch angles of the
cliff was outside the scope of this research but was investigated in Monlaur et al.
(2012) as it can strongly influence the formation and the shape of each vortex.

Comparison between the base cliff and the two triangular configurations show
that the triangular configurations display higher values of \( M_u \), which could be
misinterpreted as a better place to install wind turbines.

5. Conclusion

When an atmospheric boundary layer flow passes over generic sawtooth plan
cliffs, two delta-wing vortices are formed over each cliff top. These vortices are
strong, turbulent and quite sensitive to yaw angle; small changes in wind direction
can lead to significantly different fractional speed-up ratios and highly variable
turbulence. The fractional speed-up ratio over the cliff ranges from -1.0 to 1.0,
from a zero velocity to an increase of velocity of 100%. The complexity of the
flow field has been revealed in this unique three-dimensional study and clearly the
observed turbulence structure makes this a difficult region for taking full advantage
of potential speed-up effects associated with cliffs for wind energy considerations.
The aim of this study was to improve the understanding of the flow over sawtooth
pan cliffs, which has application for design of structures in accelerated flow regime
including wind turbines.

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